Mass spectrum and decays of charmonium states in non-relativistic quark model

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The phenomenological non-relativistic quark model (NRQM) has been employed to obtain the masses of charmonium states. In the NRQM an exhaustive study of electromagnetic decays such as radiative, leptonic and two photon decay widths have been calculated. The Hamiltonian used in the investigation has kinetic energy, confinement potential and one-gluon-exchange potential (OGEP). An overall agreement is obtained with the experimental masses and decay widths.

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I. INTRODUCTION

There is a wealth of experimental data in hadron spectroscopy that has emerged from a number of experimental facilities such as BES, E835, CLEO, BaBar, Belle, CDF, DO, NA60 etc. All these experiments are capable of discovering new hadrons, new production mechanisms, new decays and transitions and in general will be providing high precision data sample with higher confidence level. Hence the study of mass spectroscopy and decay rates of quarkonium becomes significant. Though QCD is accepted as the fundamental theory of strong interactions, there exists no exact solution to the theory in the non-perturbative low energy regime. The QCD is not exactly solvable in the non perturbative regime which is required to obtain physical properties of the hadrons. Hence various approximation methods have been employed to solve QCD in the non perturbative regime. The most promising of these is through lattice gauge theories. The lattice gauge theories involve gigantic computation hence the progress has been slow and the detailed predictions of hadron properties have not been made. As a consequence our understanding of hadrons continues to rely on insights obtained from the experiments and QCD motivated models in addition to lattice QCD results.

In the present work an attempt has been made to obtain the masses of the charmonium meson states and electromagnetic decay widths such as leptonic decay widths, two photon decay widths and radiative decay widths in the frame work of non-relativistic quark models (NRQM). This is required since several different potentials can predict the hadron spectrum but over estimate the decay rates[1–6]. Hence one needs other observables in order to test more precisely the resulting wave functions. Hence, a possibility is to study the transition between various states and their leptonic decay widths. The leptonic decay widths are a probe of the compactness of the $q\bar{q}$ system and provide important information complementary to the level spacings. For better estimations with reference to the experimental values, various corrections due to radiative processes, higher order QCD contributions are required [7]. In this context, the NRQM formalism is found to provide systematic treatment of the perturbative and non-perturbative components of QCD at the hadronic scale [8–13]. The NRQM should work better for heavy quark mesons, since a particle of mass m, localised in a volume of radius R, has a momentum $1/R$ through the uncertainty relation its kinetic energy $(< T >) << m$ only if $mR >> 1$. In the constituent quark models this is satisfied for the *c, b* and *t* quarks. Also, in NRQM the spurious excitation of the centre-of-mass (CM) motion can be eliminated easily. Hence, in heavy meson spectroscopy non relativistic models are found to be more suitable in studying the mass spectra, predicting the higher orbital states and other properties like leptonic and radiative decays.

In our previous works, we had computed the masses of *S, P* and *D* wave light wave mesons in the frame work of NRQM[14–16]. In the present work, we have considered the *S, P* and *D* wave heavy mesons with both the quark and anti quark belonging to heavy flavour sector, the charm where the total non-relativistic Hamiltonian employed has kinetic energy, confinement potential and one-gluon-exchange potential (OGEP)[17] . The aim of the present study is to obtain a minimum number of parameters which reproduce the mass spectrum, the leptonic, two photon and radiative decay widths of charmonium states. The total energy or the mass of the meson is obtained by calculating the energy eigen values of the Hamiltonian in the harmonic oscillator basis. The details about the present employed model can be found in references [14–16, 18–21].

This paper is organized as follows. In sec. 2 we briefly review the Hamiltonian used in NRQM, the non-relativistic description of leptonic, two photon and radiative decay widths. In sec. 3 we discuss the results of the calculation and the conclusions are given in section 4.

II. THEORETICAL BACKGROUND

A. The Hamiltonian

The Hamiltonian employed in our model is given by [14–16],

$$H_{NRQM} = K + V_{\rm CONF}(r_{ij}) + V_{\rm OGEP}(r_{ij}), \tag{1}$$

where

$$K = \left[\sum_{i=1}^{2} M_i + \frac{P_i^2}{2M_i}\right] - K_{\rm cm} \tag{2}$$

where M_i and P_i are the mass and momentum of the ith quark respectively. The K is the sum of the kinetic energies including the rest mass minus the kinetic energy of the centre of mass motion (CM) of the total system. The potential energy part consists of confinement term V_{CONF} and the residual interaction V_{OGEP} .

The confinement term represents the non-perturbative effect of QCD that confines quarks within the colour singlet system, and is taken to be linear [14–16].

$$V_{\rm conf}(r) = -a_c r_{ij} \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j, \tag{3}$$

where a_c is the confinement strength. Here, λ_i and λ_j are the generators of the colour SU(3) group for the i^{th} and j^{th} quark.

The following central part of two-body potential due to OGEP is employed [17],

$$V_{OGEP}(r_{ij}) = \frac{\alpha_s}{4}\ \lambda_i.\lambda_j \left[\frac{1}{r_{ij}} - \frac{\pi}{M_i M_j}\ (1 + \frac{2}{3}\ \sigma_i.\sigma_j)\ \delta(r_{ij})\right]; \tag{4}$$

where the first term represents the residual Coulomb energy and the second term the chromo-magnetic interaction leading to the hyperfine splitting. The σ_i is the Pauli spin operator and α_s the quark-gluon coupling constant.

B. Leptonic Decays

The leptonic decay width of the vector meson is by the Van Royen Weisskopf formula [22],

$$\Gamma_{(nS\to l^+l^-)} = 4\alpha^2 \langle Q^2\rangle \frac{|R_{nS}(0)|^2}{m_{nS}^2} \tag{5}$$

$$\Gamma_{(nD \to l^+l^-)} = 25\alpha^2 \langle Q^2 \rangle \frac{|R''_{nD}(0)|^2}{2m_{nD}^2 m_c^4} \quad (6)$$

In the above equations, $m_{nS}(m_{nD})$ is the mass of the VM, Q is the charge content and m_c is the mass of the quark. $R_{nS}(0)$ is the radial S wave function at the origin, and $R''_{nD}(0)$ is the second derivative of the radial D wave function at the origin. The leptonic decay width of VM including first order QCD radiative correction is given by [23, 24]

$$\Gamma'_{(nS \to l^+l^-)} = 4\alpha^2 \langle Q^2 \rangle \frac{|R_{nS}(0)|^2}{m_{nS}^2}(1 - \frac{16\alpha_s}{3\pi}) \quad (7)$$

$$\Gamma'_{(nD \to l^+l^-)} = 25\alpha^2 \langle Q^2 \rangle \frac{|R''_{nD}(0)|^2}{2m_{nD}^2 m_c^4}(1 - \frac{16\alpha_s}{3\pi}) \quad (8)$$

C. Two Photon Decays

The $c\bar{c}$ quark pair in charge conjugation even states with $J \neq 1$ can annihilate into two photons [25, 26]. For the decays of $^1S_0$, $^3P_0$ and $^3P_2$ states into two photons the expressions are [27],

$$\Gamma_{(^1S_0 \to \gamma\gamma)} = 12\alpha^2 \langle Q^2 \rangle \frac{|R_{nS}(0)|^2}{m_S^2} \quad (9)$$

$$\Gamma_{(^3P_0 \to \gamma\gamma)} = 2^4 3^3 \alpha^2 \langle Q^4 \rangle \frac{|R_{nP}(0)'|^2}{m_P^2} \quad (10)$$

$$\Gamma_{(^3P_2 \to \gamma\gamma)} = 2^6 3^2 \alpha^2 \langle Q^4 \rangle \frac{|R_{nP}(0)'|^2}{5m_P^4} \quad (11)$$

where m_p is the mass of the meson. With the first order QCD radiative correction the two photon decay rate is given by [23, 27],

$$\Gamma'_{(^1S_0 \to \gamma\gamma)} = 12\alpha^2 \langle Q^2 \rangle \frac{|R_{nS}(0)|^2}{m_S^2}[1 + \frac{\alpha_s}{\pi}(\frac{\pi^2}{3} - \frac{20}{3})] \quad (12)$$

$$\Gamma'_{(^3P_0 \to \gamma\gamma)} = 2^4 3^3 \alpha^2 \langle Q^4 \rangle \frac{|R_{nP}(0)'|^2}{m_P^2}[1 + \frac{\alpha_s}{\pi}(\frac{\pi^2}{3} - \frac{28}{9})] \quad (13)$$

$$\Gamma'_{(^3P_2 \to \gamma\gamma)} = 2^6 3^2 \alpha^2 \langle Q^4 \rangle \frac{|R_{nP}(0)'|^2}{5m_P^4}(1 - \frac{16\alpha_s}{3\pi}) \quad (14)$$

D. E1 transitions

The rate for transitions from a $^3S_1$ state to ^3P_J state [24] is given by,

$$\Gamma_{(^3S_1 \to \gamma\ ^3P_J)} = (2J+1)\frac{4}{27}e_c^2\alpha k_0^3|I_{PS}|^2 \tag{15}$$

where k_0 is the energy of the emitted photon, and I_{PS} is the radial overlap integral which has the dimension of length

$$I_{PS} = \langle P|r|S|\rangle = \int_0^\infty r^3 R_P(r)R_S(r)dr \tag{16}$$

With $R_{S,P}(r)$ being the normalised radial wave functions for the corresponding states. The transition from ^3P_J levels to a $^3S_1$ level is described by the expression for the rate

$$\Gamma_{(^3P_J \to \gamma\ ^3S_1)} = \frac{4}{9}e_c^2\alpha k_0^3|I_{SP}|^2 \tag{17}$$

For transitions $^1P_1 \to \gamma\,^1S_0$ the same expression (17) is used to calculate the rate.

E. M1 Transitions

The allowed M1 transitions are essentially $^3S_1 \to \gamma\,^1S_0$ and $^1S_0 \to \gamma\,^3S_1$. The rate for transitions from a $^3S_1$ state to $^1S_0$ state is given by,

$$\Gamma_{(n^3S_1 \to \gamma m^1S_0)} = \frac{4}{3m_c^2}e_c^2\alpha k_0^3|I_{mn}|^2 \tag{18}$$

where I_{mn} is the overlap integral for unit operator between the coordinate wave functions of the initial and the final charmonium states.

$$I_{mn} = \int_0^\infty r^2 R_{nS}(r)R_{mS}(r)dr \tag{19}$$

For transitions $^1S_0 \to \gamma\,^3S_1$ the following expression for the rate is used

$$\Gamma_{(n^1S_0 \to \gamma m\ ^3S_1)} = \frac{4}{m_c^2}e_c^2\alpha k_0^3|I_{mn}|^2 \tag{20}$$

III Fitting Procedure

There are four parameters in our model. These are the mass of charm quark *c*, the confinement strength a_c the harmonic oscillator size parameter *b* and the α_s. The values of these parameters are listed in table I. We have fixed the harmonic oscillator size parameter b using the experimental value of the square of the wave function at origin for J/ Ψ meson. The α_s is fixed by the J/Ψ- η_c splitting which comes from the colour magnetic term of OGEP. The mass of the charm quark quoted in PDG is 1270^{+70}_{-110} MeV (28). We have used in the present work a value 1160 MeV which is in the PDG mass range. The confinement strength parameter a_c is fixed by the stability condition for variation of the mass of J/Ψ against the size parameter b.

IV. RESULTS AND DISCUSSIONS

A. S wave states

The η_c *(1S)* is the lightest charmonium. Its mass has been determined through fits to the invariant mass spectrum of $\eta_c(1S)$ decay products in reactions such as $\gamma\gamma \rightarrow \eta_c(1S)$ *[29, 30]*, $B \rightarrow \eta_c(1S)K$ *[31]*, and J/ψ, $\psi(2S) \rightarrow \gamma\eta_c(1S)$ *[32, 33]* using all-charged or dominantly charged final states, and in $p\bar{p} \rightarrow \eta_c(1S) \rightarrow \gamma\gamma$ *[35, 48]*. The average value of mass of $\eta_c(1S)$ *is* 2980.4 ± 1.2 *MeV [36]* and in the latest *PDG it is* 2980.3 ± 1.2 *MeV [28]*. Our model gives the value, 2993.65 MeV (table II) which is a little higher than the experimental value [28].

The existence of $\eta_c(2S)$ was claimed by Crystal Ball collaboration [37], at a mass of 3594 ± 5 MeV. It was observed later by Belle collaboration in $B \rightarrow K(K_S K_\pi)$ [38] and in $e^+e^- \rightarrow J/\psi + X$ [39] at a significantly higher mass [35]. Study of photon-photon collisions by CLEO [29] and BaBar [30] has confirmed it. Our model calculations give significantly higher mass value for $\eta_c(2S)$ in comparison with PDG [28]. Table III and table IV give calculated values of two photon decays without and with QCD correction for η_c *(1S)* [28, 36, 40] and η_c *(2S)* [29, 41, 42] respectively. We note here that the calculated value for two-photon decay width of η_c *(2S)* agree with other theoretical predictions[41, 42] but contradicts with the experimental estimation by the CLEO collaboration [29].

The *J/ψ*, the first charmonium state discovered [43, 44] is the lowest $^3S_1$ $c\bar{c}$ state and can couple directly to virtual photons produced in e^+e^- collisions [35]. The most precise mass determination comes from the KEDR collaboration *m(J/ψ)* = 3096.917 ± 0.010 ± 0.007 MeV [45]. The calculated mass of *J/ψ* in our model is given in table II.

The $\psi(2S)$ resonance was discovered at SLAC in e^+e^- collisions [46]. The most precise $\psi(2S)$ mass measurement comes from KEDR [45]. The current world average is $m(\psi(2S)) = 3686.09 \pm 0.04$ MeV [28]. Our theoretical value is given in table II.

The leptonic J/ψ branching ratio was measured by the CLEO Collaboration by comparing the transitions $\psi(2S) \rightarrow \pi^+\pi^- J/\psi(1S) \rightarrow \pi^+\pi^- X$ with $\psi(2S) \rightarrow \pi^+\pi^- J/\psi(1S) \rightarrow \pi^+\pi^- l^+ l^- J/\psi$ [47]. The calculated values (with and without QCD correction) of leptonic decay width for J/ψ and $\psi(2S)$ are given in table VI.

B. P wave states

The $1P$ triplet states of charmonium, χ_{CJ}, were first seen in radiative decays from the $\psi(2S)$. The χ_{CJ} states lie 128/171/261 MeV below the $\psi(2S)$. Their masses can most accurately be determined in $p\bar{p}$ collisions with $\chi_{CJ} \rightarrow \gamma J/\psi \rightarrow \gamma(e^+ e^-)$ or by measuring the excitation curve, where the well known and small beam energy spread results in very low systematic uncertainty [48]. The J=0 state is wide while the J=1 and J=2 states are narrower[28]. Our calculated mass values for these states are given in the table II. We have given the calculated value of the mass of 1P singlet state of charmonium which had been observed by CLEO [49] in the same table. We have calculated two photon decay widths for $\chi_{C0}(^3P_0)$ and $\chi_{C2}(^3P_2)$ states with and without QCD correction. The calculated values are compared with those of references [36] and[50] in table V.

C. D wave states

A narrow resonance X (3872) discovered by Belle collaboration [51] in B decays has the positive C parity. An analysis of the angular correlations[52] in the process with the decay $X \rightarrow \pi^+\pi^- J/\Psi$ prefers the assignment $J^{PC} = 1^{++}$. A similar analysis by CDF [53] allows either 1^{++} or 2^{-+}. The latter assignment, however, would greatly suppress the decay of X to $D^0\overline{D}^0$, which is very close to its threshold. We have calculated the mass of $X (3872)$ taking J^{PC} as 2^{-+} which is far from good in agreement with PDG value. Hence we conclude that the possibility of $J^{PC} = 2^{-+}$ cannot be right [54]. Also positive C parity of X (3872) with decay $X \rightarrow \pi^+\pi^- J/\psi$ points to the conclusion that X resonance cannot be a pure $c\bar{c}$ system [55]. Taking $J^{PC} = 1^{--}$ we have calculated the mass of $\psi(4040)$ assigning it 1^3D_1 state which is given in table II. The leptonic decay width calculated for the same meson without QCD correction nearly agrees with the PDG value given in table VI.

D. Radiative decays

We have calculated in this paper the E1 and M1 transition widths for charmonium states. These transitions are reported in PDG. We studied the transitions allowed by long wavelength approximation. They are ${}^3P_J \rightarrow {}^3S_1$ and ${}^3S_1 \rightarrow {}^3P_J$ (E1 transitions); ${}^3S_1 \rightarrow {}^1S_0$ (M1 transitions) Along with these we have also studied a particular E1 transition corresponding to the decay ${}^1P_1 \rightarrow {}^1S_0$. In calculations energy of the photon equal to the energy difference between the resonances and in non relativistic phase space the term $\frac{Eb\,(ko)}{m_a}$ is equal to unity. Our results are shown in the tables VII,VIII, IX,X. A comparison with the ref [56] is also given. In the table X we have not included the decay $\psi(2S) \rightarrow \eta_c\gamma$ since the overlap integral vanishes for the 2S → 1S transition. This decay becomes possible due to the relativistic effects which we have not considered in this paper.

V. SUMMARY AND CONCLUSIONS

The main objective of the present work is to study the charmonium spectra and its decay properties with a single set of parameters. In this work, we have obtained the masses of Charmonium states in the frame work of NRQM in the harmonic oscillator basis spanned over a space extending up to the radial quantum number $n_{max}=4$. With a single set of parameters we get a good agreement for the masses, their leptonic decay widths, two photon decay widths and for radiative decay widths. The calculations include QCD corrections for leptonic decay widths and two photon decay widths. The NRQM formalism has the right prediction both for the spectrum and the decay rates. This work could be extended for other heavy meson systems. Work in this direction is in progress.

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TABLE I: Value of parameters

b	0.325 fm
M_c	1160 MeV
a_c	260.0 MeV fm^{-1}
α_s	0.2

TABLE II: Masses of Charmonium states (in MeV)

$^{2S+1}L_J$	Meson	Experimental Mass	Calculated mass
$^1S_0$	$\eta_c(1S)$	2980±1.2	2993.65
	$\eta_c(2S)$	3637±4	3867.32
$^3S_1$	$J/\psi(1S)$	3096.916±0.011	3053.43
	$\psi(2S)$	3686.09±0.04	3885.28
$^1P_1$	$h_c(1P)$	3525.93±0.27	3527.28
$^3P_0$	$\chi_{c0}(1P)$	3414.75± 0.31	3405.9
$^3P_1$	$\chi_{c1}(1P)$	3510.66± 0.07	3534.6
$^3P_2$	$\chi_{c2}(1P)$	3556.2± 0.09	3580.4
$^1D_2$	X(3872)	3872.2± 0.8	4063.7
$^3D_1$	$\psi(4040)$	4039± 1	4050.8

TABLE III: Two Phton decay width of $\eta_c(1S)$ (in KeV)

Meson	Ref[40]	Ref [36]	Ref[28]	$\Gamma_{(^1S_0\to\gamma\gamma)}$	$\Gamma'_{(^1S_0\to\gamma\gamma)}$
$\eta_c(1S)$	$7.0^{+1.0}_{-0.9}$	7.06 ±0.8 ± 2.3	$6.7^{+0.9}_{-0.8}$	7.89	6.2

TABLE IV: Two Phton decay width of η_c (2S) (in KeV)

Meson	Ref[29]	Ref [41]	Ref[42]	$\Gamma_{(^1S_0\rightarrow\gamma\gamma)}$	$\Gamma'_{(^1S_0\rightarrow\gamma\gamma)}$
$\eta_c(2S)$	1.3 ± 0.6	3.7	4.44 ± 0.48	4.76	3.74

TABLE V: Two Phton decay width of χ_{c0} (1P) and χ_{c2} (1P)(in KeV)

Meson	Ref[36]	Ref [50]	Γ	Γ'
$\chi_{c0}(1P)$	2.9 ± 0.4	5.32	4.15	4.19
$\chi_{c2}(1P)$	0.534 ± 0.05	0.44	0.94	0.62

TABLE VI: Leptonic decay widths of Charmonium states (in KeV)

Meson	Experimental Leptonic width[28]	Calculated Leptonic width(without QCD correction)	Calculated Leptonic width (with QCD correction)
$J/\psi(1S)$	$5.55 \pm 0.14 \pm 0.02$	5.48	3.6
$J/\psi(2S)$	2.38 ± 0.04	3.36	2.22
$\psi(4040)$	0.86 ± 0.07	0.814	0.54

TABLE VII: Decay widths for $^3S_1 \rightarrow {}^3P_J$ transitions

Transition	Expl. value $\Gamma(KeV)$	k_0(MeV)	Ref [56]	Calculated $\Gamma(KeV)$
$\psi(2S) \rightarrow \chi_{c0}(1P)\gamma$	25.76 ± 3.81	271	19.77	25.94
$\psi(2S) \rightarrow \chi_{c1}(1P)\gamma$	24.10 ± 3.49	175	38.40	20.96
$\psi(2S) \rightarrow \chi_{c2}(1P)\gamma$	21.61 ± 3.28	130	47.35	14.32

TABLE VIII: Decay widths for $^3P_J \rightarrow {}^3S_1$ transitions

Transition	Expl. value $\Gamma(KeV)$	k_0(MeV)	Ref [56]	Calculated $\Gamma(KeV)$
$\chi_{c0}(1P) \rightarrow J/\psi(1S)\gamma$	92.40 ± 41.52	318	268.67	188.63
$\chi_{c1}(1P) \rightarrow J/\psi(1S)\gamma$	240.24 ± 40.73	414	349.33	416.23
$\chi_{c2}(1P) \rightarrow J/\psi(1S)\gamma$	270.00 ± 32.78	459	387.90	567.26

TABLE IX: Decay widths for $^1P_1 \rightarrow {}^1S_0$ transitions

Transition	Expl. value $\Gamma(KeV)$	k_0(MeV)	Calculated $\Gamma(KeV)$
$h_c \rightarrow \eta_c(1S)\gamma$	seen	546	954.82

TABLE X: Decay widths for $^3S_1 \rightarrow {}^1S_0$ transitions

Transition	Expl. value $\Gamma(KeV)$	k_0(MeV)	Ref[56]	Calculated $\Gamma(KeV)$
$J/\psi(1S) \rightarrow \eta_c(1S)\gamma$	1.13 ± 0.35	117	2.04	5.14